Name KEY

MATH 181 Calculus and Analytic Geometry II Fall 2009 Actual Quiz #2

1. Find the sum of the series $\sum_{k=1}^{\infty} \frac{2^{k+1}}{5^k}$.

$$\sum_{k=1}^{\infty} \frac{2^{k+1}}{5^k} = \sum_{k=1}^{\infty} \frac{2^k \cdot 2}{5^k} = 2\sum_{k=1}^{\infty} \frac{2^k}{5^k} = 2\sum_{k=1}^{\infty}\left(\frac{2}{5}\right)^k$$

$$= 2\left[\sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^k - 1\right]$$

$$= 2\left[\frac{1}{1-2/5} - 1\right]$$

$$= 2\left(\frac{5}{3} - 1\right) = 2\left(\frac{2}{3}\right) = \frac{4}{3}$$

2. State a conclusion about the convergence of the series $\sum_{n=2}^{\infty} \frac{3n}{\sqrt{n^3-5}}$ and justify that conclusion using a comparison argument.

$\frac{3n}{\sqrt{n^3-5}} \sim \frac{3n}{\sqrt{n^3}} = 3\frac{n}{n^{3/2}} = 3\frac{1}{n^{1/2}}$ so compare with $\sum_{n}^{\infty} \frac{1}{n^{1/2}}$ which diverges since it is a p-series with $p = 1/2 \le 1$

Direct: $n^3 - 5 < n^3 \Rightarrow \sqrt{n^3-5} < \sqrt{n^3} \Rightarrow \frac{1}{\sqrt{n^3}} < \frac{1}{\sqrt{n^3-5}} \Rightarrow \frac{3n}{\sqrt{n^3}} < \frac{3n}{\sqrt{n^3-5}} \Rightarrow \frac{3}{n^{1/2}} < \frac{3n}{\sqrt{n^3-5}}$

or

Limit: $\frac{a_n}{b_n} = \frac{3n}{\sqrt{n^3-5}} \cdot \frac{n^{1/2}}{1} = \frac{3n^{3/2}}{\sqrt{n^3-5}} \to 3$

Either (direct or limit) tells us that $\sum^{\infty} \frac{1}{n^{1/2}}$ and $\sum^{\infty} \frac{3n}{\sqrt{n^3-5}}$ have same behavior

Since $\sum^{\infty} \frac{1}{n^{1/2}}$ diverges, we conclude that $\sum^{\infty} \frac{3n}{\sqrt{n^3-5}}$ diverges.

3. Use the ratio test to reach a conclusion about the convergence of the series $\sum_{k}^{\infty} \frac{4^{k+1}}{k!}$.

$$\frac{a_{k+1}}{a_k} = \frac{4^{k+2}}{(k+1)!} \cdot \frac{k!}{4^{k+1}} = \frac{k!}{(k+1)!} \cdot \frac{4^{k+2}}{4^{k+1}} = \frac{1}{k+1} \cdot \frac{4}{1} = \frac{4}{k+1} \to 0 < 1.$$

Since $\frac{a_{k+1}}{a_k}$ converges to a limit less than 1, we conclude that $\sum^{\infty} \frac{4^{k+1}}{k!}$ converges by the ratio test.